Received February 12, 1767.

XVIII. *A Computation of the Diſtance of the Sun from the Earth.* *By* S. Horſley, *LL. B. Rector of* Saint Mary, Newington, *in* Surry, *F. R. S.*

Read March 26, 1767.

I Offer the following computation, rather as a verification than an amendment of Dr. Stewart's. The method, in which I have purſued, is different from what is uſed by that great and able geometrician, in his treatiſe on the diſtance of the ſun, but founded entirely on the theorems eſtabliſhed in that and the preceding tracts of the ſame author.

Let TA be a given line. Take A*m*, ſo that TA may be to A*m*, as the moon's accelerating attraction to the earth, to the ſun's mean diſturbance of that attraction. Take AG quintuple of A*m*. Take AP, ſuch that twice A*m* may be to AP, as TG to TA. Now it is proved in the twenty-fifth propoſition of Dr. Stewart's fourth tract, that the cube of TA is to the cube of TP, in the duplicate proportion of the periodic month to the anomaliſtic month. Therefore the proportion of TA^3 to TP^3, and conſequently that of TA to TP, is given; and by diviſion, that of TA to AP is given. Therefore TA being given, AP is given. Now $TG : TA = 2Am : AP$. That is,

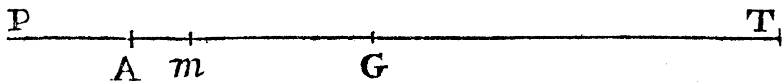
 TA

$TA - 5\,Am : TA = 2\,Am : AP$. Therefore $TA \times 2\,Am = \overline{TA - 5\,Am} \times AP$. Therefore $2\,Am = \frac{\overline{TA \times AP} - \overline{5\,Am \times AP}}{TA}$. That is $2\,Am + \frac{\overline{5\,Am \times AP}}{TA}$, or $\frac{\overline{2\,TA + 5\,AP} \times Am}{TA} = AP$.

That is,

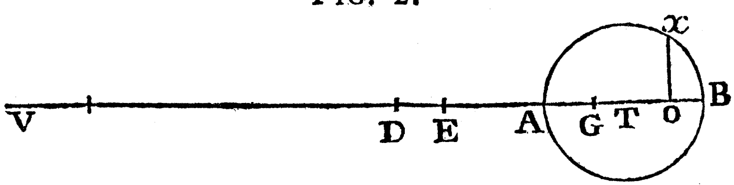
FIG. 1.

$Am = \frac{TA \times AP}{2TA + 5AP}$.



2. Let TA represent the moon's mean diſtance from the earth. Take TV, ſuch that TA may be to TV, in the duplicate proportion of the periodic month to the ſidereal year. Take TG, ſuch that VT may

FIG. 2.



be to TG in the proportion of the moon's accelerating attraction to the earth, to the ſun's mean diſturbance of that attraction. Take TE, ſuch that TE may be to TA, as TA to TG. Take EO, ſuch that the rectangle EOA may be equal to $3TE \times TA$. Upon the centre T, with the interval TA, deſcribe a circle. Draw Ox perpendicular to AB, meeting the circle in x. Take AD = AT. The proportion of TA to TV being given, and TA being given, TV is given. But the proportion of TV to TG is given. Therefore TG is given, and the proportion of TG to TA is given. $TG : TA = TA : TE$. Therefore the proportion of TA to TE is given. Therefore TE is given. Therefore $3TE \times TA$ is given. Therefore $EO \times OA$ is given. And EA ($= TE - TA$) is given. Therefore AO is given. But AB ($= 2AT$) is given. Therefore OB is given. Therefore $AO \times OB$ is given. $AO \times OB = Ox^2$ (by the circle). Therefore Ox^2, and conſequently Ox is given. But DB ($= 3AT$) is given. Therefore the proportion of DB to Ox is given.

And

And the proportion of DB to Ox, is that of the mean diſtance of the ſun, to the mean diſtance of the moon.

This is in brief the method of my computation. The computation is as follows:

The periodic month is to the anomaliſtic month, as 57600 to 58091.
Therefore (in Fig. 1.) $TA^3 : TP^3 = 57600^2 : 58091^2 = 3317760000 : 3374564281$

$$\frac{3374564281}{3317760000} = 1{,}017121274896315\dot{5}864197530\dot{8}64197530, \text{ \&c.}$$

Hence, by extracting the cube root, I find $TA : TP = 1 : 1{,}005674827053$.
Therefore put $TA = 1$. Then $TP = 1{,}005674827053$; and $AP = 0{,}005674827053$.

$$\text{Hence } \frac{TA \times AP}{2TA + 5AP} = 0{,}002797722 = Am.$$

(See Fig. 2). The ſquare of the periodic month is to the ſquare of the ſidereal year, as 1 to 178,725.
Therefore $TA : TV = 1 : 178{,}725$.
But $TV : TG = 1 : 0{,}002797722$.
Therefore $TA : TG = 1 : 178{,}725 \times 0{,}002797722 = 1 : 0{,}50002286445$.
$TA : TG = TE : TA$. Therefore $TE : TA = 1 : 0{,}50002286445$.

Therefore put $TE = 1$.
Then $TA = 0{,}50002286445$
And $EA = 0{,}49997713555$
And $3TE \times TA = 1{,}50006859335 = EOA$.

$$\text{Hence } AO \left(= \sqrt{TE \times TA + \frac{EA^2}{4}} - \frac{EA}{2} \right) = 1{,}00003658292.$$

But $AB = 2TA = 1{,}00004572890$
Therefore $OB = 0{,}00000914598$
Therefore $AO \times OB = 0{,}000009146314586654661 6$
Therefore $\sqrt{AO \times OB} = 0{,}00{,}32024287 = Ox$
But $DB = 3TA = 1{,}500068593$
Hence $DB : Ox = 496{,}0073 : 1$.

These

Theſe computations have been made with no ſmall rigor. I was ſenſible that, to obtain an accurate concluſion, it was neceſſary to determine AO with extreme preciſion; and for that purpoſe I ſubmitted to the laborious taſk of computing the foregoing numbers to the 11th or 12th decimal place, by the common operations of arithmetic. In the reſult I differ from Dr. Stewart, by much leſs than $\frac{1}{6000}$th part of the whole diſtance, that is, by leſs than 5 ſemidiameters of the earth; a very contemptible difference in ſo nice a calculation. That great mathematician indeed ſeems to have flattered himſelf, that he had determined the ſun's diſtance within $\frac{1}{4360000}$ of the truth. I ſuſpect that when he affirmed this, he did not conſider that to attain ſo great an accuracy in the concluſion, the line Eq in his method (vide Stewart on the ſun's diſtance, Fig. 10.), or AO in mine, ſhould be determined ſtrictly to the 11th or 12th decimal place. And after the utmoſt rigor of computation, I am afraid any pretenſions to ſuch extreme nicety in the reſult will be but ill-founded. For it is very likely that theſe computations repreſent the ſun's diſtance leſs than it really is: becauſe the whole progreſſion of the moon's apogee (which is the baſis of the calculation) is aſcribed to the ſun's diſturbance of the moon's gravitation to the earth. Whereas part of it muſt be due to the diſturbances of the planets. What part is due to them we cannot tell, and therefore cannot allow for it. But in giving the whole to the ſun we certainly overrate his diſturbing force, and by that means muſt obtain too ſmall a diſtance.

It

It is moſt likely indeed, that the motion of the apogee produced by the diſturbing forces of the planets bears but a very ſmall, perhaps inſenſible, proportion to the whole. But thoſe who are maſters of Dr. Stewart's theorems will eaſily perceive that an inſenſible error in the proportion of the moon's gravity, and the ſun's diſturbance, may produce a very ſenſible error in the proportion of the mean diſtances. And therefore the real diſtance is probably greater by two or three ſemidiameters of the earth than theſe computations make it.

This, however, is much too nice a point for the approaching tranſit, or, perhaps, for any method of obſervation, to determine. The higheſt expectations f aſtronomers will be anſwered, if they can come within 50 or 60 ſemidiameters of the earth.

It is to be hoped, that every civilized nation of the univerſe will give due attention to that intereſting phænomenon, which we, the preſent poſſeſſors of theſe ſublunary regions, ſhall behold no more; and that proper perſons will be ſent in due time, and duly equipped, to the moſt advantageous ſtations.

If the deciſions of obſervation in ſo nice a point ſhould be found to agree with the previous concluſions of theory, the diſciples of Newton will have no ſmall reaſon to exult in a new atteſtation of nature, to the truth of their great maſter's doctrine.

But it is much to be wiſhed, that they, who ſhall be deputed to proſecute this curious ſearch, in diſtant and ſequeſtered parts, may diveſt themſelves of all prejudice; that they may have nothing at heart, but, that which the world will expect from them, the

the advancement of real ſcience; that they may be diligent in their obſervations, and faithful in their reports; and not ſacrifice the repoſe of their own minds, or the intereſts of philoſophy, to the credit of an admired hypotheſis, the memory of a friend, or the jealouſies of rival nations.

If the moon's mean diſtance from the earth be $60\frac{1}{3}$ ſemidiameters of the earth, the ſun's mean diſtance is 30008,4416 ſemidiameters of the earth.

The ſun's ſemidiameter is to the ſemidiameter of the earth, as 139,876 to 1. The globe of the ſun is to the globe of the earth, as 2736718,8 to 1; and the ſun's horizontal parallax is $6'' \; 52''',415$.

February 8.

To ſatisfy myſelf more fully of the accuracy of my work, I this day re-computed the whole, from the determination of EA, in Dr. Stewart's approximating method. I found the proportion of DB to vs (ſee Dr. Stewart on the diſtance of the ſun, Fig. 10.), that of 496,00579 to 1; and the proportion of DB to tv, that of 496,00805 to 1. The mean of theſe two gives the proportion of DB to ox, nearly that of 496,0069 to 1. Which differs from the reſult of my former computation by leſs than $\frac{1}{1240000}$ of the whole; and the method of the former computation is undoubtedly the moſt accurate.

Supplement

Supplement to the foregoing Paper.

Read June 19, 1767.

IN deducing the diſtance of the ſun in ſemidiameters of the earth, and his horizontal parallax, from the proportion above concluded between the ſun's mean diſtance and that of the moon; I have ſuppoſed the latter to be $60\frac{1}{2}$ ſemidiameters of the earth, as it is reckoned by Sir Iſaac Newton. According to the hypotheſis which ſeems to be now generally received, that the denſity of the moon is very nearly equal to that of the earth, (the French reckon it rather leſs), the moon's mean diſtance ſhould be little more than 60,23207, that is, not quite $60\frac{1}{4}$ ſemidiameters of the earth. But from ſome computations that I have formed with great care; I have reaſon to think, that Sir Iſaac Newton's determination is much nearer to the truth; that the denſity of the moon is actually greater than that of the earth, in the proportion of 6 to 5 nearly; and that the moon's mean diſtance amounts to 60,441 ſemidiameters of the earth; which differs from the diſtance aſſigned by Sir Iſaac Newton, by leſs than $\frac{1}{1007}$ of the whole.

S. Horſley.